Radiative Energy Loss and the Electrical Conductivity in Nonequilibrium MHD Generators

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Abstract

THE radiative energy loss and its effect on the electrical conductivity in a nonequilibrium Faraday magnetohydrodynamic generator have been studied. Considering the non-Maxwellian distribution of electrons in a nonequilibrium argon-potassium plasma with significant radiation loss, the calculated electrical conductivity is found to show the experimentally observed peculiar behavior of decreasing conductivity in a certain current density range.

Contents

The electrical conductivity of a nonequilibrium magnetohydrodynamic (MHD) plasma is often determined by means of the Saha equation at the electron temperature, where the population of the excited states is assumed to be in thermal equilibrium with the electrons having a Maxwellian distribution. This consideration is, however, strictly justified only when the collisional loss due to electron/atom elastic collisions is the dominant electron energy loss. When the radiative energy loss arising from inelastic collisions of electrons with heavy particles in seeded inert gases is significant, the electron distribution function is in general non-Maxwellian. With a view to explaining the observed hump[1-3] in the conductivity/current relation as a peculiar behavior, Sakao and Sato[1] considered the non-Maxwellian distribution of electrons in the presence of radiative energy loss. Without explicitly considering the radiative energy loss, they considered a parameter characterizing such loss and calculated the electrical conductivity showing the qualitative decrease in a certain range of current density.

The present paper deals with the radiative energy loss by following Lutz[4] and then calculating the electrical conductivity of a nonequilibrium argon-potassium plasma using a non-Maxwellian electron distribution similar to Sakao and Sato.[1] When the radiative loss is negligible, the electrons follow the Maxwellian velocity distribution defined by one electron temperature. Since inelastic collisions giving rise to radiation loss remove electrons from the high-energy tail of the electron velocity distribution, the electrons are redistributed quite differently with tail being depressed below its Maxwellian value. This depressed high-energy tail is represented as if there were a new Maxwellian at lower electron temperature denoted by T_{eh}, which is used in the Saha equation to determine the electron number density n_e. As the electron mobility μ_e depends only on the elastic collisions of electrons with other heavy particles, it is calculated on the basis of electron temperature $T_{e\ell}$ characterizing the bulk of the electrons taking part in elastic collisions.

The electron energy equation determining the electron temperature in the steady state is given as[5]

$$J^2/\sigma = \dot{\Omega} + \dot{R} \tag{1}$$

where J is the Faraday density, σ the electrical conductivity, $\dot{\Omega}$ the collisional loss, and $\dot{R}$ the radiative energy loss. The collisional loss arising due to elastic collisions of electrons with heavy particles in a potassium-argon plasma is given by

$$\dot{\Omega} = 3\delta k(T_e - T_g)n_e\nu_e m_e/2m_A \tag{2}$$

where T_e is the electron temperature, T_g the gas temperature, m_A the mass of argon atom, k the Boltzmann's constant, and $\delta(=2)$ the collisional loss parameter. The effective electron collision frequency ν_e in a potassium-argon plasma is given as[5]

$$\nu_e = \frac{4}{3}\left(\frac{8kT_e}{\pi m_e}\right)^{1/2}(n_A\bar{Q}_A + n_k\bar{Q}_k + n_e\bar{Q}_i) \tag{3}$$

where n_A is the argon number desity, n_k the potassium number density, and $\bar{Q}$ the averaged energy collision cross section for the electron and heavy-particle collisions. The radiative energy loss in a potassium-argon plasma is approximated as[4]

$$\dot{R} = \sum_j \frac{4c}{d}(dn_k)^{1/2}(f_j\Delta\nu_j)^{1/2}[B_{\nu j}(T_e) - B_{\nu j}(T_g)] \tag{4}$$

where $B_{\nu j}$ is the Planck function, $\Delta\nu_j$ the collision broadening, f_j the oscillator strength, d the distance between two opposite electrodes, and c a constant equal to 4.84×10^{-3} $m\cdot s^{-1/2}$. In the calculation of R, the summation of j is carried out for two potassium resonance lines of 7665 and 7699 Å. It is important to note here that Tong et al.[6] have used c equal to 1.93×10^{-2} $m\cdot s^{-1/2}$, which is about four times larger than the above value obtained from Lutz.[4] The collisional broadening is expressed as[7]

$$\Delta\nu_j = n_A\sigma_j^2\left[\frac{8kT_e}{\pi}\left(\frac{1}{m_k} + \frac{1}{m_A}\right)\right]^{1/2} \tag{5}$$

where σ_j^2 is the optical collision cross section of jth potassium line and m_k the mass of the potassium atom.

As the high-energy electrons only are responsible for inelastic collisions, T_{eh} is obtained by solving the full energy equation (1) with the help of Eqs. (2-5). The electron temperature $T_{e\ell}$ is evaluated by neglecting $\dot{R}$ in Eq. (1). This consideration is quite justified as most of the joule heat is

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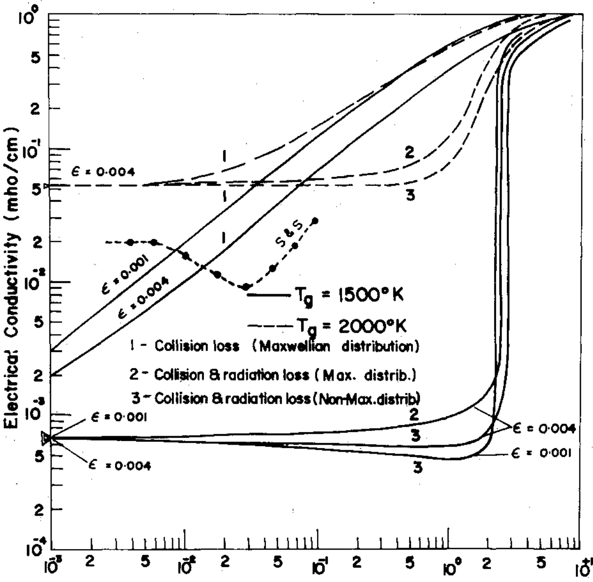

Fig. 1 Electrical conductivity vs current density. Curve denoted by S&S (Sakao and Sato[1]) shows the experimental σ-J relation for atmospheric argon with 0.013% potassium at $T_g = 1773$ K.

given to low-energy electrons, which do not take part in inelastic collisions. From n_e and μ_e, the electrical conductivity of a nonequilibrium plasma has been calculated. For the sake of comparison, we have also calculated σ for Maxwellian distributions with associated electron temperatures equal to $T_{e\ell}$ and T_{eh}.

As a typical illustration of our results, we have considered the potassium-seeded argon plasma at 1 atm pressure as the working gas flowing in a Faraday MHD channel with $d = 0.013$ m (similar to the cylindrical channel of Zukoski et al.[8]). The average electron/atom collision cross sections[8] for argon and potassium are 7.0×10^{-21} and 4.0×10^{-18} m^2, respectively. The average electron/ion collision cross section $\bar{Q}_i$ has been taken as 4.0×10^{-16} m^2. The optical collision cross sections[9] of potassium lines have been taken as 57.7×10^{-20} and 60.4×10^{-20} m^2 with corresponding oscillator strengths f_j equal to 0.682 and 0.332.

First, we have calculated the radiative energy loss using Eq. (4) for different $T_{e\ell}$ values and found them in agreement with those of Zukoski et al.[8] Then, we have calculated σ for different values of J considering Maxwellian and non-Maxwellian electron velocity distributions at different T_g and seed fractions (Fig. 1). The corresponding σ in the thermal equilibrium (i.e., $T_{e\ell} = T_{eh} = T_g$) has been shown in Fig. 1 by open triangles at the ordinate axis. As a typical example of the dip in σ observed in the experiments,[1-3] the data of Sakao and Sato[1] (S&S) is shown in Fig. 1 for comparison. It is well known that the maximum σ is obtained at the optimum value of seed fraction ϵ. The seed fraction of 0.1% being close to the optimum value gives rise to higher equilibrium σ than that of 0.4% at a particular T_g. Curves labeled with 1 in Fig. 1 show σ calculated by assuming the energy loss as the collisional loss only and the Maxwellian distribution at $T_{e\ell}$. Curves 2 show σ for Maxwellian distribution at T_{eh}, incorporating collisional and radiative energy losses. Curves 3 are for the case of the non-Maxwellian electron distribution, accounting for the collisional and radiative energy losses.

Curve 1 in Fig. 1 depicts the marked increase of σ over its equilibrium value, whereas a very slight increase in curve 2 and practically same or slight decrease in curve 3 are seen for the values of J below 1 A/cm^2. Curve 3 maintains the conductivity at its equilibrium value up to 1A/cm^2 for $T_g = 2000$ K and $\epsilon = 0.4\%$, but is shows a decreasing conductivity for current densitites below 0.4 A/cm^2 at $T_g = 1500$ K. This decrease in σ is even more for $\epsilon = 0.1\%$ at $T_g = 1500$ K and reaches about 34.0%. It is thus evident that curve 1 or 2 cannot explain in any case observed decrease in σ for current densities below 1 A/cm^2. The observed peculiar behavior of the σ-J relation, however, may be seen qualitatively in curve 3, where σ is plotted based on the present analysis taking account of radiation loss with a non-Maxwellian distribution. Quantitative explanation of the experimental σ-J relation is not attempted here because of different channel and complex electrode geometries employed in the experiments. The dip in σ for the S&S curve occurs earlier compared to our results as the current density for the experimental results has been calculated by dividing the current with the cross-sectonal area, which is much larger than the actual electrode area. Curves 1-3 yield practically the same conductivity after 5 A/cm^2, indicating that the consideration of equilibrium at the electron temperature without radiation is sufficient for the analysis of σ of a nonequilibrium plasma for current densitites larger than 5 A/cm^2.

Thus, it is concluded that, when the radiative energy loss is more than or comparable to the collisional loss, the non-Maxwellian distribution of electrons with a proper radiative energy loss estimation is the most accurate consideration for the analysis of the electrical conductivity of a nonequilibrium plasma.

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References

[1]Sakao, F. and Sato, H., "Nonequilibrium Electrical Conductivity of a Potassium-Seeded Argon Plasma," *Physics of Fluids*, Vol. 12, Oct. 1969, pp. 2063-2071.

[2]Kerrebrock, J.L. and Hoffman, M.A., "Non-Equilibrium Ionization due to Electron Heating: II. Experiments," *AIAA Journal*, Vol. 2, June 1964, pp. 1080-1087.

[3]Kerrebrock, J.L. and Dethlefsen, R., "Experimental Investigation of Fluctuations in a Nonequilibrium MHD Plasma," *AIAA Journal*, Vol. 6 Nov. 1968, pp. 2115-2121.

[4]Lutz, M.A., "Radiation and Its Effect on the Nonequilibrium Properties of a Seeded Plasma," *AIAA Journal*, Vol. 5, Aug. 1967, pp. 1416-1423.

[5]Lengyel, L.L., "Two-Dimensional Current Distributions in Faraday Type MHD Energy Converters Operating in the Non-Equilibrium Conduction Mode," *Energy Conversion*, Vol. 9, Jan.-March 1969, pp. 13-23.

[6]Tong, J.Z., Harada, N., Shimizu, M., Tahara, M., Kabashima, S., Yamasaki, H., and Shioda, S., "Electrical Discharge in a Nonequilibrium Slightly Seeded MHD Plasma," *Energy Conversion and Management*, Vol. 23, Jan.-Feb., 1983, pp. 11-21.

[7]Onda, K., Kaga, Y. and Kato, K., "Measurement of MHD Combustion—Gas Temperatures and Potassium Number Densities in the Presence of Cold Boundary Layers," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 26, Aug. 1981, pp. 147-156.

[8]Zukoski, E.E., Cool, T.A. and Gibson, E.G., "Experiments Concerning Non-equilibrium Conducting in a Seeded Plasma," *AIAA Journal*, Vol. 2, Aug. 1964, pp. 1410-1417.

[9]Hofmann, F.W. and Kohn, H., "Optical Cross Section of Resonance Lines Emitted by Flames under Conditions of Partial Thermal Ionization," *Journal of the Optical Society of America*, Vol. 51, May 1961, pp. 512-521.